

# PROJECT DOCUMENTATION REPORT

**ON**

**SOLVING Map Coloring Problem using CSP**

**in the context of USA and Australia maps.**

**PROGRAMMING PROJECT 3**

**ITCS 6150 - Intelligent Systems**

Project Guidance By

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**MAP COLORING PROBLEM STATEMENT:-**

**INTRODUCTION TO CSP:**

A CSP is a problem composed of a finite set of variables each of which has a finite domain of values and a set of constraints. Each constraint is defined over some subset of the original set of variables and restricts the values these variables can simultaneously take. The task is to find an assignment of a value for each variable such that the assignments satisfy all the constraints In some problems the goal is to find all such assignments. Constraint satisfaction problems on finite domains are typically solved using a form of search. The most used techniques are variants of backtracking, constraint propagation, and local search.

**GRAPH COLORING:**

Given a graph G = (V, E) and an integer k, a k-coloring of G is a one-one mapping of vertices to colors, such that adjacent vertices are assigned to different colors. The Minimum Graph Coloring Problem (Min–GCP) consists in finding the minimum k such that a k-coloring exists. Such minimum k is known as the chromatic number of G and is denoted by χ(G), or simply by χ. Min–GCP is NP-hard. The chromatic number is bounded from below by the size of the maximum clique of G, known as the clique number ω(G) which is equal to χ(G) when G is a perfect graph. Chromatic number is the minimum number of colors required to satisfy the constraint that adjacent vertices do not have same color.

**MAP COLORING:**

The Map coloring problem is similar to the graph colouring problem. In map coloring the constraint is that states which are adjacent to each other ie share a border should not have the same color. In the case for map colouring of Austrailia the chromatic number is 3 meaning if number of colors used is less than 3 then we would have some states which cannot be assigned colors as their domain size would reduce to 0. In the case of USA map colouring chromatic number is 4.

**MAP COLORING SOLUTION APPROACHES:**

Thereare 3 approaches we have used in this project DFS, DFS with forward checking, DFS with forward checking and propagation through singleton domain.

**Depth first search (BACKTRACKING):**

**T**he concept in backtracking applied to map coloring is that once any variables domain reaches 0 then the algorithm goes back to previous states and see if using other options in the domain would yield a color for the one with the empty set in its domain. Every time the algorithm checks the next state only after reaching there, there are no prechecks done.

**DFS WITH FORWARD CHECKING:-**

The concept here is exactly same as backtracking only that next check is pre-checked making the algorithm smarter than Just BACKTRACKING. Number of backtracks are significantly reduced.

**DFS WITH FORWARD CHECKING AND PROPAGATION THROUGH SINGLETON DOMAINS:-**

Here the algorithmchecks among all possibilities of next states and choses the one with domain value equal to 1 and propagates to the next unassigned variables from the one with domain =1. Number of backtracks are further reduced and the algorithm is relatively faster.

**HEURISTICS USED:-**

There may be a number of options or nodes to choose from the next states. Any one of the states may be chosen at random and we could progress the map colouring algorithm. With using heuristics we get an order of chosing the variables depending on some factors. Some of the most commonly used heuristics are as follows below.

1.) **MINIMUM REMAINING VALUES:-**

In this heuristic propagation follows in the order of those nodes with least number of values in its domain . With respect to map coloring problem If one state has 2 permissible values in its domain and another state has 3 then the state with 2 values would be chosen first , here permissible refers to reducing domain size because of constraints imposed that adjacent states cannot have same color.

2.) **DEGREE HEURISTIC:-**

The idea here is assign a value to the variable that is involved in the largest number of constraints on other unassigned variables. It is often used as a means to reduce the number of same next possibilities ie as a tie breaker with Minimum remaining values heuristic to chose the best next when all next nodes have the same number of domain values after a variable assignment is done using Mrv.

3.) **LEAST CONSTRAINING VALUE:-**

Here the chosen heuristic rules out the fewest values in the remaining variables.

Variables used:-

List\_of\_colors:- a list used to store list of colors

Backtracks:-to store the total number of backtracks

Start\_time and end\_

time to calculate total run duration

Numcolors:-chromatic number

Statedict:- list of states in usa or aus

root: stores color of root node.

Mystatedict:-color assigned as program procedes

**SIMULATION RESULTS FOR USA:-**

**FOR DFS**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | Time taken |
| 1 | 1012451 | 21.79387936592102seconds |
| 2 | 1012451 | 18.7117121219635seconds |
| 3 | 1012451 | 19.913596630096436seconds |

**RESULT SAMPLE:-**

VERIFIED ANSWER

(1, {'Maine': 'red', 'Minnesota': 'red', 'South Dakota': 'blue', 'Illinois': 'red', 'Utah': 'red', 'Wyoming': 'green', 'Texas': 'blue', 'Idaho': 'blue', 'Wisconsin': 'blue', 'Connecticut': 'red', 'Pennsylvania': 'red', 'Kansas': 'green', 'West Virginia': 'blue', 'North Carolina': 'green', 'Colorado': 'blue', 'California': 'red', 'Florida': 'red', 'Vermont': 'red', 'Virginia': 'red', 'North Dakota': 'green', 'Michigan': 'green', 'New Jersey': 'blue', 'Nevada': 'green', 'Arkansas': 'green', 'Mississippi': 'red', 'Iowa': 'green', 'Kentucky': 'green', 'Maryland': 'green', 'Louisiana': 'black', 'Alabama': 'green', 'Oklahoma': 'red', 'New Mexico': 'green', 'Rhode Island': 'blue', 'Massachusetts': 'green', 'South Carolina': 'red', 'Indiana': 'blue', 'Delaware': 'black', 'Tennessee': 'blue', 'Georgia': 'black', 'Arizona': 'black', 'Nebraska': 'red', 'Missouri': 'black', 'New Hampshire': 'blue', 'Ohio': 'black', 'Oregon': 'black', 'Washington': 'red', 'Montana': 'red', 'New York': 'black'})

NUMBER OF BACKTRACKS: 1012451

TIME OF EXECUTION: 19.74671196937561seconds

**DFS WITH FORWARD CHECK**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | Time taken |
| 1 | 10238 | 1.145329475402832seconds |
| 2 | 10238 | 0.16092705726623535seconds |
| 3 | 10238 | 0.15390944480895996seconds |

RESULT SAMPLE:-

VERIFIED ANSWER

(1, {'New Hampshire': 'red', 'Oklahoma': 'red', 'Tennessee': 'red', 'Illinois': 'red', 'New Mexico': 'blue', 'Kentucky': 'blue', 'West Virginia': 'green', 'Maryland': 'red', 'Maine': 'blue', 'Wisconsin': 'blue', 'Missouri': 'green', 'Minnesota': 'red', 'Montana': 'red', 'Massachusetts': 'blue', 'South Carolina': 'red', 'North Dakota': 'blue', 'Pennsylvania': 'blue', 'Arizona': 'green', 'South Dakota': 'green', 'Ohio': 'red', 'Oregon': 'red', 'Alabama': 'blue', 'Indiana': 'green', 'Rhode Island': 'red', 'Virginia': 'black', 'Idaho': 'green', 'Nevada': 'blue', 'Nebraska': 'red', 'New York': 'green', 'Utah': 'red', 'Michigan': 'black', 'Kansas': 'blue', 'Florida': 'red', 'Connecticut': 'black', 'Iowa': 'black', 'Wyoming': 'blue', 'Louisiana': 'red', 'California': 'black', 'Vermont': 'black', 'Texas': 'green', 'Georgia': 'green', 'New Jersey': 'red', 'North Carolina': 'blue', 'Washington': 'blue', 'Delaware': 'green', 'Colorado': 'black', 'Mississippi': 'green', 'Arkansas': 'blue'})

NUMBER OF BACKTRACKS: 10238

TIME OF EXECUTION: 0.15990805625915527seconds

10286

**DFS WITH SINGLETON AND FORWARD CHECK:-**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | Time taken |
| 1 | 10238 | 1.145329475402832seconds |
| 2 | 10238 | 0.16092705726623535seconds |
| 3 | 10238 | 0.15390944480895996seconds |

**EXECUTION RESULTS WITH HEURISTICS FORWARD CHECK WITH DFS:-**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | **Time taken** |
| 1 | 1713 | 0.09692907333374023seconds |
| 2 | 1713 | 0.16092705726623535seconds |
| 3 | 1713 | 0.15390944480895996seconds |

RESULT SAMPLE:-

(1, {'Illinois': 'red', 'Oklahoma': 'red', 'California': 'red', 'Utah': 'red', 'Wyoming': 'blue', 'Missouri': 'blue', 'Michigan': 'green', 'Texas': 'blue', 'Iowa': 'green', 'Delaware': 'red', 'Tennessee': 'red', 'Maryland': 'blue', 'Kentucky': 'green', 'Montana': 'red', 'Minnesota': 'red', 'Connecticut': 'red', 'Louisiana': 'red', 'West Virginia': 'red', 'Pennsylvania': 'green', 'Nebraska': 'red', 'Kansas': 'green', 'Indiana': 'blue', 'Rhode Island': 'blue', 'Arizona': 'blue', 'Florida': 'red', 'Massachusetts': 'green', 'South Dakota': 'black', 'Nevada': 'green', 'South Carolina': 'red', 'Ohio': 'black', 'New Hampshire': 'red', 'Idaho': 'black', 'Washington': 'red', 'Colorado': 'black', 'Oregon': 'blue', 'New Jersey': 'blue', 'Mississippi': 'blue', 'Arkansas': 'green', 'Vermont': 'blue', 'Wisconsin': 'blue', 'Alabama': 'green', 'Georgia': 'blue', 'Maine': 'blue', 'New Mexico': 'green', 'North Carolina': 'green', 'New York': 'black', 'Virginia': 'black', 'North Dakota': 'blue'})

NUMBER OF BACKTRACKS: 1713

TIME OF EXECUTION: 0.09692907333374023seconds

**HEURISTICS FOR DFS:-**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | **Time taken** |
| 1 | 701287 | 48.621222734451294seconds |
| 2 | 701287 | 48.621222734451294seconds |
| 3 | 701287 | 48.621222734451294seconds |

**Singleton With Heuristic:**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | **Time taken** |
| 1 | 9691 | 0.45360498428344727seconds |
| 2 | 9691 | 0.41360498428344727seconds |
| 3 | 9691 | 0.41360498428344727seconds |

**TABULATION FOR AUSTRAILIA:-**

**DFS:-**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | **Time taken** |
| 1 | 0 | 4.9591064453125e-05seconds |
| 2 | 0 | 3.9591064453125e-05seconds |
| 3 | 0 | 2.9591064453125e-05seconds |

**WITH HEURISTIC:-**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | **Time taken** |
| 1 | 0 | 4.9591066153125e-05seconds |
| 2 | 0 | 1.9581164453125e-05seconds |
| 3 | 0 | 2.9691064853125e-05seconds |

**DFS WITH FORWARD CHECK:-**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | **Time taken** |
| 1 | 0 | 2.9591064254345e-05seconds |
| 2 | 0 | 3.9591064453455e-05seconds |
| 3 | 0 | 2.9591064452355e-05seconds |

**WITH HEURISTIC:-**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | **Time taken** |
| 1 | 0 | 4.9591076153125e-05seconds |
| 2 | 0 | 1.9581134453125e-05seconds |
| 3 | 0 | 2.9691024853125e-05seconds |

**DFS WITH SINGLETON AND FORWARD CHECK:-**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | **Time taken** |
| 1 | 0 | 2.9591064453340e-05seconds |
| 2 | 0 | 3.9591064453467e-05seconds |
| 3 | 0 | 2.9591064453695e-05seconds |

**WITH HEURISTIC:-**

|  |  |  |
| --- | --- | --- |
| Number of run | Number of backtrack | **Time taken** |
| 1 | 0 | 4.9591066153125e-05seconds |
| 2 | 0 | 1.9581164453125e-05seconds |
| 3 | 0 | 2.9691064853125e-05seconds |

SAMPLE OUTPUTS FOR USA and visualization













